Problems with Histogram Equalization

Image: Large concentration of dark gray-level pixels
FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).
Histogram Specification/Matching

- **Histogram equalization method:**
  - *Only generates one result: an image with approximately uniform histogram (without any flexibility)*
  - *Enhancement may not be achieved as desired*

- **Histogram specification:**
  - *Transform an image according to a specified gray-level histogram*

- **Includes**
  - *Specify particular histogram shapes (p_z(z)) capable of highlighting certain gray-level ranges*
  - *Obtain the transformation function for transformation of r to z*
Histogram Specification

- Define a random variable \( z \) such that
  - \( z = G^{-1}(s) \)
  - \( z = G^{-1}(T(r)) \)

- **Histogram Specification**
  - Apply HE on \( r \) to obtain \( s \)
  - Apply inverse of cdf of \( z \) on \( s \)
Histogram Specification

- **Step 1**: Equalize the levels of the original image
- **Step 2**: Specify the desired pdf and obtain the transformation function
- **Step 3**: Apply the inverse transformation function to the levels obtained in step 1
Step 1: Perform Histogram Equalization

<table>
<thead>
<tr>
<th>$r_k$</th>
<th>$n_k$</th>
<th>$p_r(r_k) = n_k/MN$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0 = 0$</td>
<td>790</td>
<td>0.19</td>
</tr>
<tr>
<td>$r_1 = 1$</td>
<td>1023</td>
<td>0.25</td>
</tr>
<tr>
<td>$r_2 = 2$</td>
<td>850</td>
<td>0.21</td>
</tr>
<tr>
<td>$r_3 = 3$</td>
<td>656</td>
<td>0.16</td>
</tr>
<tr>
<td>$r_4 = 4$</td>
<td>329</td>
<td>0.08</td>
</tr>
<tr>
<td>$r_5 = 5$</td>
<td>245</td>
<td>0.06</td>
</tr>
<tr>
<td>$r_6 = 6$</td>
<td>122</td>
<td>0.03</td>
</tr>
<tr>
<td>$r_7 = 7$</td>
<td>81</td>
<td>0.02</td>
</tr>
</tbody>
</table>

$p_r(r_k)$

$s_k$

$p_s(s_k)$
Step 2: Specify the Desired PDF and Get the CDF

\[ p_z(z_q) \]

\[ G(z_q) \]

<table>
<thead>
<tr>
<th>( z_q )</th>
<th>( G(z_q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

**TABLE 3.3**

All possible values of the transformation function \( G \) scaled, rounded, and ordered with respect to \( z \).
Step 3: Apply the Inverse PDF

Find the smallest value of $z_q$ so that $G(z_q)$ is closest to $s_k$.

<table>
<thead>
<tr>
<th>$s_k$</th>
<th>$z_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

TABLE 3.4
Mappings of all the values of $s_k$ into corresponding values of $z_q$. 
Histogram Specification

$p_r(r_k)$ vs $r_k$

$p_z(z_q)$ vs $z_q$

$r_z(z_q)$ vs $z_q$
Histogram Specification Example
Issues with Histogram specification/matching:

- No rule for specifying an optimal histogram
- Each given enhancement task needs to be analyzed on a case-by-case basis
- Histogram specification is somehow a trial-and-error process
Local Histogram Processing

- The histogram processing methods mentioned up to now are global transformation where:
  - Function is designed according to the gray-level distribution over an entire image
  - Global transformation methods may not be suitable for enhancing details over small areas
  - Where number of pixels in these small areas may have negligible influence on designing the global transformation function
Local Histogram Processing

- To enhance details over small areas in an image

Procedure

- Define a neighborhood (e.g. N8)
- Move it from pixel to pixel.
- For every pixel
  - Histogram computed for the neighborhood
  - Transfer function computed for HE or H Spec
  - Applied on Centre Pixel
Local HE for 3x3 Neighborhood

**FIGURE 3.26** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size $3 \times 3$. 
Using Histogram Statistics for Image Enhancement

- Mean gives the average brightness of the image
- Variance ($\sigma^2$) and its square root the standard deviation gives the deviation of intensities on average from the mean value (average contrast)
- Global statistics

\[
m = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)
\]

\[
\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)
\]

\[
\sigma^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[ f(x, y) - m \right]^2
\]
Using Histogram Statistics

- **Local Statistics:**

  - $S_{xy}$: a neighborhood (subimage) of specific size centered at $(x,y)$

  $$m_{s_{xy}} = \sum_{i=0}^{L-1} r_i p_{s_{xy}}(r_i)$$

  $$\sigma_{s_{xy}}^2 = \sum_{i=0}^{L-1} \left[ r_i - m_{s_{xy}} \right]^2 p_{s_{xy}}(r_i)$$
Local Enhancement using Histogram Statistics

- The statistical parameters can be used in various ways
- Enhance the background filament
- Enhance details in dark areas while leaving light area unchanged.
- Define rules to choose the candidate pixels that need to be enhanced
Local Enhancement using Histogram Statistics

- A pixel at point \((x,y)\) is considered if:
  - \(m_{s_{xy}} \leq k_0M_G\), where \(k0\) is a positive constant less than 1.0, and \(M_G\) is global mean
  - \(\sigma_{s_{xy}} \leq k_2D_G\), where \(D_G\) is the global standard deviation and \(k_2\) is a positive constant
  - Also need to put a lower limit on SD to avoid distorting areas which don’t have details, i.e., \(k_1D_G \leq \sigma_{s_{xy}}\), with \(k1 < k2\)

- A pixel that meets all above conditions is processed simply by multiplying it by a specified constant, \(E\), to increase or decrease the value of its gray level relative to the rest of the image.

- The values of pixels that do not meet the enhancement conditions are left unchanged.

\[
g(x, y) = \begin{cases} 
E \cdot f(x, y) & \text{if } m_{s_{xy}} \leq k_0M_G \text{ AND } k_1D_D \leq \sigma_{s_{xy}} \leq k_2D_G \\
 f(x, y) & \text{otherwise}
\end{cases}
\]
FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)